$$R = \text{radius} = \frac{AC}{2\sin b} = \frac{1}{4}\sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{(s-a)(s-b)(s-c)(s-d)}}$$

$$2R = D = \tfrac{1}{2}\sqrt{\frac{(39)(42)(38)}{(4)(6)(3)(5)}} = \tfrac{1}{2}\sqrt{172.9} = 6.57457225.$$

Also solved by *P. S. BERG, H. C. WHITAKER, J. R. BALDWIN, P. H. PHILBRICK, J. W. SCHEFFER.*

[Note.—The formula for the area of an inscriptable quadrilateral is $A = \sqrt{s(s-a)(s-b)(s-b)(s-c)(s-d)}$, where $s = \frac{1}{2}(a+b+c+d)$.—Ed.]

7. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in the Ohio University, Athens, Ohio.

Through each point of the straight line $x = my + h$ is drawn a chord of the parabola $y^2 = 4ax$, which is bisected in the point. Prove that this chord touches the parabola $(y+2am)^2 = 8a(x-h)$.

II. Solution by L. E. PRATT, Tecumseh, Nebraska.

Let $y = Bx + C \ldots (1)$ be a straight line cutting the given parabola. The co-ordinates of the middle point of the chord intercepted by the curve are

$$\left(\frac{2a-BC}{B^2},\ \frac{2a}{B}\right).$$

Substituting these for x and y in the equations of the given straight line, we have

$$BC = 2a - 2amB - hB^2 \ldots (2).$$

If (x_1, y_1) be any point of (1) we have $y_1 = Bx_1 + C \ldots (3)$.

Eliminating C from (2) and (3) we obtain a quadratic in B which may be written

$$B = \frac{y_1 + 2am \pm \sqrt{(y_1+2am)^2 - 8a(x_1-h)}}{2(x_1-h)} \quad \ldots (4).$$

This result shows that two chords bisected by the straight line $x = my + h$ may be drawn through the point x_1, y_1; that when the roots of B, the angular coefficient, are equal the two chords coincide in one; that this takes place when the radical in (4) is equal to zero. But when the radical equals zero the point x_1, y_1, is a point of the parabola

$$(y+2am)^2 = 8a(x-h)$$

and the straight line (1) is tangent to it.

This problem was solved in a very excellent manner by *Professors HUME, SCHEFFER,* and *ZERR.*

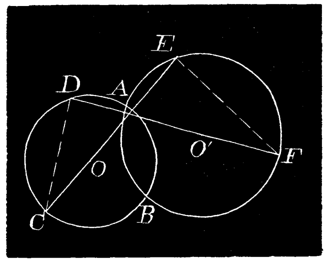
9. Proposed by J. C. GREGG, Superintendent of Schools, Brazil, Indiana.

Two circles intersect in A and B. Through A two lines CAE and DAF are drawn, each passing through a centre and terminated by the circumference. Show that $CA \times AE = DA \times AF$. [*Euclid.*]

Solution by Miss GRACE H. GRIDLEY, Student in Kidder Institute, Kidder, Missouri.

Let the straight lines CAE and DAF pass through the point of intersection A of the two circles, and through the centers O and O', respectively.

Connect by straight lines the points D and C, and E and F. Then the triangles CDA and AED are right triangles, being inscribed in a semi-circle. The two triangles are also similar, having the acute angle EAF of the one equal to the acute angle DAC of the other.



$\therefore CA : AF :: AD : AE$, the homologous sides of similar triangles being proportional.

$\therefore CA\times AE=AF\times AD$. Q. E. D.

Remark.—When the angle CAF is a right angle AE and AD each equal zero. When the angle CAF is less than a right angle, the point E will fall on the semi-circumference ABF and the point D will fall on the diameter AF and $CA\times AE=AF\times AD$ as before.

Also solved by *J. F. W. SCHEFFER, JOSIAH H. DRUMMOND, ROBERT J. ALEY, G. B. M. ZERR, J. A CALDERHEAD, P. S BERG.* and *P. H. PHILBRICK.*

10. **Proposed by ERIC DOOLITTLE, Instructor in Mathematics, State University of Iowa, Iowa City.**

If MN be any plane, and A and B any points without the plane, to find a point P, in the plane, such that $AP+PB$ shall be a minimum.

Solution by THADDEUS MERRIMAN, South Bethlehem, Pennsylvania.

First, suppose the points to be on opposite sides of the plane; the point where the straight line joining the given points pierces the plane is the required point P; since a straight line is the shortest distance between two points.

Second, suppose the points to be on the same side of the plane; let AB be the straight line joining them, CD the projection of AB on the plane MN, and AC the perpendicular which projects A on the plane. Now, produce AC to E making $AC=CE$, and join E and B; the point P where EB cuts CD is the required point. For, join A and P, and let Q be any other point in the plane; join A and Q, and B and Q, also Q and E. Now, since $AC=CE$ by construction, $AP=PE$ and $AQ=QE$; consequently $BQ+QE=BQ+QA$, and therefore, since $BE<BQ+QE$, we have $AP+PB<AQ+QB$, or $AP+PB$ is a minimum.

[This demonstration is by Thaddeus Merriman, the 17 year old son of Professor Mansfield Merriman.—Ed.]

Also solved by *J. H. BEACH, G. B. M. ZERR, LEONARD E. DICKSON, F. A. SWANGER, H. C. WHITAKER, P. H. PHILBRICK, A. H. BELL* and *J. F. W. SCHEFFER.*

PROBLEMS.

31. **Proposed by Professor G. I. Hopkins, Manchester, New Hampshire.**

A field is bounded as follows: N. 14° W. 15.2 chains; N. 70°30′ E. 20.43 chains; S. 6° E. 22.79 chains; N. 86° 30′ W. 18 chains. A spring within it bears from the second corner S. 75° E. 7.9 chains. It is required to cut off 10 acres from the west side of the field by a straight fence through the spring. How far will it be from the first corner to the point at which the division fence meets the fourth side?